Rezime

Za razliku od hiperboličkih tačaka ekvilibrijuma, gdje je lokalna stabilnost obuhvaćena kroz nekoliko teorema i gdje uvijek imamo jasnu situaciju, ispitivanje stabilnosti nehiperboličkog ekvilibrijuma u diferentnim jednadžbama i diskretnim dinamičkim sistemima je specifično i zahtijeva dodatna ispitivanja i upotrebu različitih tehnika i metoda. Ovaj rad posvećuje pažnju upravo nehiperboličkom ekvilibrijumu kod diferentnih jednadžbi drugog reda, te jednodimezionalnih i dvodimenzionalnih diskretnih dinamičkih sistema.

Prvi dio rada (*Stabilnost nehiperboličkog ekvilibrijuma diferentne jednadžbe prvog reda*) obuhvata diferentnu jednadžbu prvog reda, to jest, jednadžbu oblika

$$x_{n+1} = f(x_n), \quad n = 0, 1, ...,$$

koja ujedno predstavlja i jednodimenzionalni diskretni dinamički sistem.

Na samom početku su definisani osnovni pojmovi, te je uveden pojam stabilnosti tačaka ekvilibrijuma, a zatim je posebna pažnja posvećena stabilnosti nehiperbiličkih tačaka ekvilibrijuma. Nehiperbolički ekvilibrijum se pojavljuje u slučaju kada je $|f'(\bar{x})| = 1$, pri čemu je $\bar{x}$ tačka ekvilibrijuma navedene diferentne jednadžbe prvog reda, odnosno, fiksna tačka preslikavanja f. Razmatrana su dva slučaja i to kada je $f'(\bar{x}) = 1$ i $f'(\bar{x}) = -1$, te su navedeni kriteriji za utvrđivanje stabilnosti nehiperboličkog ekvilibrijuma u oba slučaja.

Drugi dio rada(*Stabilnost nehiperboličkog ekvilibrijuma dvodimenzionalnih diskretnih dinamičkih sistema*) razmatran je dvodimenzionalni diskretni dinamički sistem:

$$\left.\begin{array}{rcl} x_{n+1} & = & f(x_n, y_n) \\ y_{n+1} & = & g(x_n, y_n) \end{array}\right\} n = 0, 1, ...,$$

odnosno, diferentna jednadžba drugog reda:

$$x_{n+1} = f(x_n, y_{n-1}), \quad n = 0, 1, ...$$

Uvedeni su osnovni pojmovi o stabilnosti, analogni onima u prvom poglavlju, a zatim je navedeno nekoliko metoda za ispitivanje stabilnosti nehiperboličkih tačaka ekvilibrijuma (to su one tačke u kojima Jakobijan matrica pridruženog lineariziranog sistema ima bar jednu svojstvenu vrijednost po modulu jednaku jedan). Tako je opisan metod Lyapunovljeve funkcije, metod invarijante, metod centralne mnogostrukosti, KAM metod, Neimark-Sackerova bifurkacija, te metod monotonih preslikavanja. Svaki od ovih metoda ilustrovan je pomoću odgovarajućih, pažljivo odabranih, primjera.

Summary

Unlike hyperbolic equilibrium points, where local stability is included through several theorems and where we always have a clear situation, the stability analysis of non-hyperbolic equilibrium points in differential equations and discreete dynamical systems is specific and requires additional tests and the use of different techniques and methods. This paper devotes attention precisely at the non-hyperbolic equilibrium of the differential equations and discreete dynamical systems.

First part(*Stability of non-hyperbolic equilibrium of the first-order differential equation*) include first-order differential equation, i.e. equation of the form

$$x_{n+1} = f(x_n), \quad n = 0, 1, ...$$

At the begining, the basic concept are defined and we introduce the concept of an equilibrium point and then special attention is paid to the stability of non-hyperbolic equilibrium points, i.e., in the case when $|f'(\bar{x})| = 1$, where $\bar{x}$ is an equilibrium point of listed first-order differential equation or fixed point of map f. Two cases are considered, when $f'(\bar{x}) = 1$ and when $f'(\bar{x}) = -1$, and the criteria for determining stability of non-hyperbolic equilibrium in both cases are listed.

In the second part(*Stability of non-hyperbolic equilibrium of the two-dimensional discrete dynamical systems*) it was discussed two-dimensional discrete dynamical system:

$$\left.\begin{array}{rcl} x_{n+1} & = & f(x_n, y_n) \\ y_{n+1} & = & g(x_n, y_n) \end{array}\right\} n = 0, 1, ...,$$

or second-order differential equation:

$$x_{n+1} = f(x_n, y_{n-1}), \quad n = 0, 1, ...$$

Basic concepts of stability are introduced, analogous to those in the first chapter, and then several methods for investigating the stability of non-hyperbolic equilibrium points (these are the points where the Jacobian matrix of the associated linearized system has at least one eigenvalue per modulo equal to one) are listed. So, the Lyapunov function method, the invariant method, the Method of center manifold, the KAM method, the Neimark-Sacker bifurcation, and the Method of monotone maps are described. Each of these methods is illustrated with an appropriate, carefully selected, example.